

The Commutative and Associative Properties

The **commutative property of multiplication** ($a \times b = b \times a$) says that when two numbers are multiplied, changing the order of the factors does not change the product; for example, $1/6 \times -2/3 = -2/3 \times 1/6$.

The **associative property of multiplication** says that $(a \times b) \times c = a \times (b \times c)$. This property allows us to multiply a sequence of more than two numbers. Because only two numbers at one time can be multiplied, we use the associative property to associate the two numbers we want to multiply at each step. No matter which two numbers we multiply, the product will be the same; for example, $(1/2 \times 2/3) \times 3/4 = 1/2 \times (2/3 \times 3/4)$.

The commutative and associative properties are often used in sequence to simplify multiplication of more than two numbers. The following example explains how the two properties are used to simplify $\frac{3}{4} \times \frac{1}{5} \times \frac{1}{6}$.

		Simplification Strategy
$\frac{3}{4} \times \frac{1}{5} \times \frac{1}{6}$		
1	$\frac{3}{4} \times \frac{1}{6} \times \frac{1}{5}$	The commutative property is used to switch $\frac{1}{5}$ and $\frac{1}{6}$ so that $\frac{3}{4}$ and $\frac{1}{6}$ can be associated.
2	$(\frac{3}{4} \times \frac{1}{6}) \times \frac{1}{5}$	The associative property is used to associate $\frac{3}{4}$ and $\frac{1}{6}$ so they can be multiplied.
3	$\frac{1}{8} \times \frac{1}{5}$	The result is two factors instead of the original three.
4	$\frac{1}{40}$	Multiplying the two factors gives the product.

Game Description and Materials

Make It Easy is a game for two players that uses the commutative and associative properties to give students mental math practice multiplying rational numbers. The game emphasizes the logic of step-by-step procedures. For each three-term multiplication expression, players compete to have their Game Cards fill all the spaces whose steps lead to a single, simplified number.

Game materials include one Game Board, Game Cards, and a spinner.

The **object of the game** is to fill more columns than the other player.

Game Board

- The **playing portion** of the Game Board consists of four rows and five columns.
- The **column headings** show the three-addend expressions for players to simplify. The strategy is to associate the two addends that are easier to add. In this case, the strategy is to associate the addends whose denominators are the same.
- The **row headings** identify the steps in the solution and show examples.

	$-3 - \frac{1}{2} + 2$	$\frac{2}{3} - \frac{1}{4} - \frac{1}{3}$	$1.5 - \frac{3}{5} - 6.5$	$-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	$3.5 - 6 + 2.25$	$\frac{5}{2} - \frac{1}{3} + \frac{1}{2}$
	Make it easy $-3 - \frac{1}{2} + 2$	Make it easy $\frac{2}{3} - \frac{1}{4} - \frac{1}{3}$	Make it easy $1.5 - \frac{3}{5} - 6.5$	Make it easy $-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	Make it easy $3.5 - 6 + 2.25$	Make it easy $\frac{5}{2} - \frac{1}{3} + \frac{1}{2}$
1	Commutative Property $-3 + 2 - \frac{1}{2}$	Commutative Property	Commutative property	Commutative property	Commutative property	commutative property
2	Associative property $(-3+2) - \frac{1}{2}$	Associative property	Associative property	Associative property	Associative property	Associative property
3	Two addends $-1 - \frac{1}{2}$	Two addends	Two addends	Two addends	Two addends	Two addends
4	Sum $-1\frac{1}{2}$	Sum	Sum	Sum	Sum	Sum

- The top portion of the Game Board shows the three-term expressions and , under it, the “Make-It Easy” approach to simplification. In each case, the two outer terms are bolded to “hint” that the commutative property should be used to place the terms next to each other so they can be associated and added.

$-3 - \frac{1}{2} + 2$	$\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	$1.5 - \frac{3}{5} - 6.5$	$-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	$3.5 - 6 + 2.25$	$5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$
Make it easy	Make it easy	Make it easy	Make it easy	Make it easy	Make it easy
$-3 - \frac{1}{2} + 2$	$\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	$1.5 - \frac{3}{5} - 6.5$	$-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	$3.5 - 6 + 2.25$	$5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$

Cards

There are two sets of Game Cards, one for each player. One set is shaded; the other is not shaded. Except for the shading, the sets are identical. Each Game Card shows one of the four steps for simplifying a three-addend expression. These example Game Cards show simplification steps for the expression $\frac{1}{5} + 4 + \frac{3}{5}$:

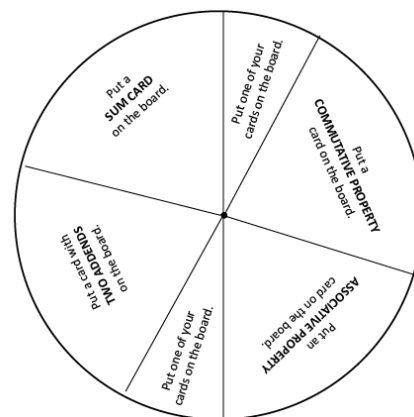
- Step 1: Commutative Property Cards** show the original three addends, but the order of two addends has been switched. In this example, $\frac{1}{5}$ and 4 have been switched so that $\frac{1}{5}$ and $\frac{3}{5}$ can be associated and added.
- Step 2: Associative Property Cards** show two of the three addends being associated so they can be added.
- Step 3: Two-Addend Cards** show the results of adding the associated addends.
- Step 4: Sum Cards** show the sums.

$4 + \frac{1}{5} + \frac{3}{5}$	$4 + \frac{1}{5} + \frac{3}{5}$
$4 + \left(\frac{1}{5} + \frac{3}{5}\right)$	$4 + \left(\frac{1}{5} + \frac{3}{5}\right)$
$4 + \frac{4}{5}$	$4 + \frac{4}{5}$
$4\frac{4}{5}$	$4\frac{4}{5}$

Spinner

Players use the spinner to identify their next move.

- The spinner may land on one of the four solution steps: commutative property, associative property, two addends, or sum.
- The spinner may land on one of two Chance options: “Put one of your cards on the board” or “Remove one of your cards (and all cards under it) from the board.” Cards removed from the board are given back to the player to be played at a later



Getting Ready to Play

Players place the Game Board between them. Each player gets a set of light or dark Game Cards. Players place their cards on the table in front of them in any organization that makes sense to them.

Let's Play!

1. Players take turns spinning and placing cards on the Game Board according to the spinner's "directions." Several rules apply:

- Cards must be placed *in order* in each column:
 1. Commutative property
 2. Associative property
 3. Two Addends
 4. Sum
- A player may play a Game Card ONLY when that step is the next unfilled space in a simplification sequence.
- Any gray or white card may be placed in a space below any other gray or white card.
- Players must state the purpose of each simplification step they take; for example,

1. "I will use the commutative property to switch ____ and ____ so I can associate them to make ____."
2. "I will use the associative property to group ____ and ____ so I can add them to make ____."
3. "I will add ____ and ____ so I have only ____ and ____ left to add."
4. "I will add ____ and ____ to get a sum of ____."

Players who neglect to state aloud the purpose of a step must remove their card from the Game Board and lose their turn.

2. Play continues until all the expressions have been simplified (cards fill every space in every column).
3. The game is over when all the spaces have been filled.

Scoring

Players earn 10 points for each column filled by their own cards and 1 point for each additional space on the board filled by their own card.

The winner is the player with more points.



$a + b + c = d$
 Associative Property of Addition $(a + b) + c = d$
 $a + (b + c) = d$
 Commutative Property of Addition: $a + b = b + a$

Example

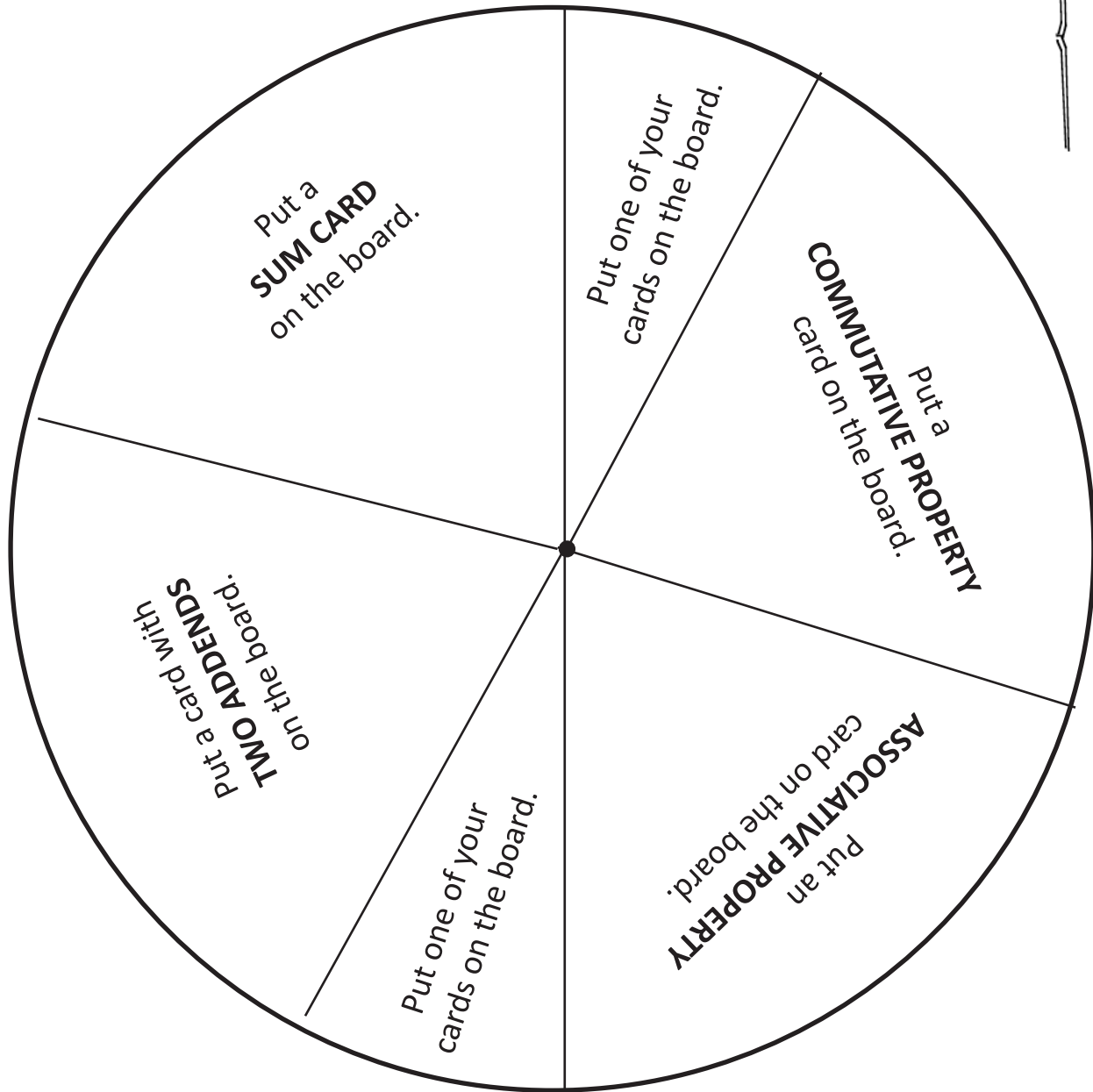
$-3 - \frac{1}{2} + 2$	$\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	$1.5 - \frac{3}{5} - 6.5$	$-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	$3.5 - 6 + 2.25$	$5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$
Make it easy $-3 - \frac{1}{2} + 2$	Make it easy $\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	Make it easy $1.5 - \frac{3}{5} - 6.5$	Make it easy $-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	Make it easy $3.5 - 6 + 2.25$	Make it easy $5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$
Commutative Property $-3 + 2 - \frac{1}{2}$	Commutative Property $\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	Commutative property $1.5 - \frac{3}{5} - 6.5$	Commutative property $-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	Commutative property $3.5 - 6 + 2.25$	Commutative property $5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$
Associative property $(-3 + 2) - \frac{1}{2}$	Associative property $\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	Associative property $1.5 - \frac{3}{5} - 6.5$	Associative property $-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	Associative property $3.5 - 6 + 2.25$	Associative property $5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$
Two addends $-1 - \frac{1}{2}$	Two addends $\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	Two addends $1.5 - \frac{3}{5} - 6.5$	Two addends $-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	Two addends $3.5 - 6 + 2.25$	Two addends $5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$
Sum $-1\frac{1}{2}$	Sum $\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	Sum $1.5 - \frac{3}{5} - 6.5$	Sum $-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	Sum $3.5 - 6 + 2.25$	Sum $5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$
1					
2					
3					
4					

Communicating the purpose of each step

1. "I will use the commutative property to switch ___ and ___ so I can associate them to make ___"
2. "I will use the associative property to group ___ and ___ so I can add them to make ___"
3. "I will add ___ and ___ so I have only ___ and ___ left to add."
4. "I will add ___ and ___ to get a sum of ___."

$\frac{1}{4} + \frac{2}{3} - \frac{1}{3}$	$1.5 - 6.5 - \frac{3}{5}$	$\frac{5}{12} - \frac{1}{3} - \frac{3}{4}$	$3.5 + 2.25 - 6$	$-\frac{1}{3} + 5\frac{1}{2} + 3\frac{1}{2}$
$\frac{1}{4} + \left(\frac{2}{3} - \frac{1}{3}\right)$	$(1.5 - 6.5) - \frac{3}{5}$	$\frac{5}{12} + \left(-\frac{1}{3} - \frac{3}{4}\right)$	$(3.5 + 2.25) - 6$	$-\frac{1}{3} + \left(5\frac{1}{2} + 3\frac{1}{2}\right)$
$\frac{1}{4} + \frac{1}{3}$	$-5 - \frac{3}{5}$	$\frac{5}{12} - \frac{13}{12}$	$5.75 - 6$	$-\frac{1}{3} + 9$
$\frac{7}{12}$	$-5\frac{3}{5}$	$-\frac{2}{3}$	$-.25$	$8\frac{2}{3}$
$\frac{1}{4} + \frac{2}{3} - \frac{1}{3}$	$1.5 - 6.5 - \frac{3}{5}$	$\frac{5}{12} - \frac{1}{3} - \frac{3}{4}$	$3.5 + 2.25 - 6$	$-\frac{1}{3} + 5\frac{1}{2} + 3\frac{1}{2}$
$\frac{1}{4} + \left(\frac{2}{3} - \frac{1}{3}\right)$	$(1.5 - 6.5) - \frac{3}{5}$	$\frac{5}{12} + \left(-\frac{1}{3} - \frac{3}{4}\right)$	$(3.5 + 2.25) - 6$	$-\frac{1}{3} + \left(5\frac{1}{2} + 3\frac{1}{2}\right)$
$\frac{1}{4} + \frac{1}{3}$	$-5 - \frac{3}{5}$	$\frac{5}{12} - \frac{13}{12}$	$5.75 - 6$	$-\frac{1}{3} + 9$
$\frac{7}{12}$	$-5\frac{3}{5}$	$-\frac{2}{3}$	$-.25$	$8\frac{2}{3}$

Use a pencil to anchor the looped end of a small paper clip to the center of the circle. Spin the paper clip.



$a + b + c = d$
 Associative Property of Addition $(a + b) + c = d$
 $a + (b + c) = d$
 Commutative Property of Addition: $a + b = b + a$

Example

Expression $-3 - \frac{1}{2} + 2$	$\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	$1.5 - \frac{3}{5} - 6.5$	$-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	$3.5 - 6 + 2.25$	$5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$
Make it easy $-3 - \frac{1}{2} + 2$	Make it easy $\frac{2}{3} + \frac{1}{4} - \frac{1}{3}$	Make it easy $1.5 - \frac{3}{5} - 6.5$	Make it easy $-\frac{1}{3} + \frac{5}{12} - \frac{3}{4}$	Make it easy $3.5 - 6 + 2.25$	Make it easy $5\frac{1}{2} - \frac{1}{3} + 3\frac{1}{2}$
Commutative Property $-3 + 2 - \frac{1}{2}$	Commutative Property $\frac{1}{4} + \frac{2}{3} - \frac{1}{3}$	Commutative property $1.5 - 6.5 - \frac{3}{5}$	Commutative property $\frac{5}{12} - \frac{1}{3} - \frac{3}{4}$	Commutative property $3.5 + 2.25 - 6$	Commutative property $-\frac{1}{3} + 5\frac{1}{2} + 3\frac{1}{2}$
Associative property $(-3 + 2) - \frac{1}{2}$	Associative property $\frac{1}{4} + \left(\frac{2}{3} - \frac{1}{3}\right)$	Associative property $(1.5 - 6.5) - \frac{3}{5}$	Associative property $\frac{5}{12} + \left(-\frac{1}{3} - \frac{3}{4}\right)$	Associative property $(3.5 + 2.25) - 6$	Associative property $-\frac{1}{3} + \left(5\frac{1}{2} + 3\frac{1}{2}\right)$
Two addends $-1 - \frac{1}{2}$	Two addends $\frac{1}{4} + \frac{1}{3}$	Two addends $-5 - \frac{3}{5}$	Two addends $\frac{5}{12} - \frac{13}{12}$	Two addends $5.75 - 6$	Two addends $-\frac{1}{3} + 9$
Sum $-1\frac{1}{2}$	Sum $\frac{7}{12}$	Sum $-5\frac{3}{5}$	Sum $-\frac{2}{3}$	Sum -0.25	Sum $8\frac{2}{3}$

1. "I will use the commutative property to switch ___ and ___ so I can associate them to make ___"
2. "I will use the associative property to group ___ and ___ so I can add them to make ___"
3. "I will add ___ and ___ so I have only ___ left to add.
4. "I will add ___ and ___ to get a sum of ___."

$a + b + c = d$ Commutative Property of Addition: $a + b = b + a$
 Associative Property of Addition $(a + b) + c = d$
 $a + (b + c) = d$

Example

Expression $-3 - \frac{1}{2} + 2$	Make it easy $-3 - \frac{1}{2} + 2$	Make it easy	Make it easy	Make it easy	Make it easy
Commutative Property $-3 + 2 - \frac{1}{2}$	Commutative Property $-3 + 2 - \frac{1}{2}$	Commutative property	Commutative property	Commutative property	Commutative property
Associative property $(-3 + 2) - \frac{1}{2}$	Associative property $(-3 + 2) - \frac{1}{2}$	Associative property	Associative property	Associative property	Associative property
Two addends $-1 - \frac{1}{2}$	Two addends $-1 - \frac{1}{2}$	Two addends	Two addends	Two addends	Two addends
Sum $-1\frac{1}{2}$	Sum $-1\frac{1}{2}$	Sum	Sum	Sum	Sum

1. "I will use the commutative property to switch ___ and ___ so I can associate them to make ___"
2. "I will use the associative property to group ___ and ___ so I can add them to make ___"
3. "I will add ___ and ___ so I have only ___ and ___ left to add.
4. "I will add ___ and ___ to get a sum of ___."

