

Game Description and Materials

Property Power is the culminating game in the Serious Play series of games. It is here that players discover the power of the operations properties in providing rationale for steps in procedures that simplify numerical (and algebraic) expressions. Teams of two players compete to record the properties used to justify each step.

Game materials include Justification Cards that show the steps taken to simplify ten numerical expressions of varying complexity. Players need a marker (not included) to record their steps.

The **object of the game** is to earn more points than the other team by assigning properties to simplification procedures.

Justification Cards

The Justification Cards consist of ten simplification sequences. Each sequence shows increasingly simple equivalent expressions leading from the given statement in step 1 to the one-number-only last step in which numerical expressions have been simplified in one-step-at-a-time procedures. A space to the right of each step is left for players to record the operations property (or calculation) that provides the rationale for the step. Depending on its complexity, each simplification sequence has a point value of 3, 4, or 5. (Justification Cards should be laminated so that they can be used repeatedly.)

A properties “cheat sheet” is provided for each team.

ADDITION PROPERTIES		MULTIPLICATION PROPERTIES	
Identity +	$a + 0 = a$	Identity x	$a \times 1 = a$
Inverse +	$a + ^-a = 0$	Inverse x	$a \times \frac{1}{a} = 1$
Commutative +	$a + b = b + a$	Commutative x	$a \times b = b \times a$
Associative +	$(a + b) + c = a + (b + c)$	Associative x	$(a \times b) \times c = a \times (b \times c)$
Definition of subtraction	$a - b = a + ^-b$	Definition of division	$a \div b = a \times \frac{1}{b}$
Distributive property of multiplication over addition			

4 - 3(7 - 14)	Reasons	3
1. 4 - 3(7 - 14)	1. Given	
2. 4 - 21 + 42	1-2.	
3. 4 + ^-21 + 42	2-3.	
4. 4 + (^-21 + 42)	3-4.	
5. 4 + 21	4-5.	
6. 25	5-6.	

$2\left(\frac{1}{6} + 4\right) \div 8$	Reasons	4
1. $2\left(\frac{1}{6} + 4\right) \div 8$	1. Given	
2. $2\left(\frac{1}{6} + 4\right) \times \frac{1}{8}$	1-2.	
3. $\frac{1}{3} + 8 \times \frac{1}{8}$	2-3.	
4. $\frac{1}{3} + \left(8 \times \frac{1}{8}\right)$	3-4.	
5. $\frac{1}{3} + 1$	4-5.	
6. $\frac{1}{3}$	5-6.	

$3 \times 10 \times \frac{1}{3} - 8$	Reasons	5
1. $3 \times 10 \times \frac{1}{3} - 8$	1. Given	
2. $3 \times \frac{1}{3} \times 10 - 8$	1-2.	
3. $\left(3 \times \frac{1}{3}\right) \times 10 - 8$	2-3.	
4. $1 \times 10 - 8$	3-4.	
5. $(1 \times 10) - 8$	4-5.	
6. $10 - 8$	5-6.	
7. $10 + ^-8$	6-7.	
8. 2	7-8.	

Getting Ready to Play

Four players arrange themselves in two teams of two. Each team takes a Properties Chart and reviews the operations properties they will use in the game. They place the Justification Cards face up on the table so that the cards are visible and accessible to both teams.

ADDITION PROPERTIES		MULTIPLICATION PROPERTIES	
Identity +	$a+0=a$	Identity x	$a \times 1 = a$
Inverse +	$a+^{-}a=0$	Inverse x	$a \times \frac{1}{a} = 1$
Commutative +	$a+b=b+a$	Commutative x	$a \times b = b \times a$
Associative +	$(a+b)+c = a+(b+c)$	Associative x	$(a \times b) \times c = a \times (b \times c)$
Definition of subtraction	$a- b = a+^{-}b$	Definition of division	$a \div b = a \times \frac{1}{b}$
Distributive property of multiplication over addition		$a(b+c) = ab+ac$	

Let's Play!

- Teams look over the Justification Cards to judge the difficulty of the various simplification sequences.
- Each team chooses a card to start with. Using their Properties Chart to help them determine which property is being used as justification for each step, the players on each team work together to complete the steps on their chosen card.
 - They identify each property by name followed by a "+" or "x," when needed, depending on whether the property describes addition or multiplication.
 - Some steps involve addition or multiplication calculations rather than properties. Players write "addition" or "multiplication" to explain those steps.
- Each time a team completes a card, the players select another card and continue identifying the properties.
- The game continues in this manner until all the cards have been completed.

The example shows the one-property-at-a-time procedure.

$\frac{2}{3}\left(3+\frac{5}{2}\right)-2 \div \frac{5}{3}+3$	Reasons
1. $\frac{2}{3}\left(3+\frac{5}{2}\right)-2 \div \frac{5}{3}+3$	1. Given
2. $\frac{2}{3}\left(3+\frac{5}{2}\right)-2 \times \frac{3}{5}+3$	1-2. Definition of division
3. $2+\frac{5}{3}-2 \times \frac{3}{5}+3$	2-3. Distributive
4. $2+\frac{5}{3}+^{-}2 \times \frac{3}{5}+3$	3-4. Definition of subtraction
5. $2+^{-}2+\frac{5}{3} \times \frac{3}{5}+3$	4-5. Commutative +
6. $(2+^{-}2)+\frac{5}{3} \times \frac{3}{5}+3$	5-6. Associative +
7. $0+\frac{5}{3} \times \frac{3}{5}+3$	6-7. Inverse +
8. $0+\left(\frac{5}{3} \times \frac{3}{5}\right)+3$	7-8. Associative x
9. $0+1+3$	8-9. Inverse x
10. $(0+1)+3$	9-10. Associative +
11. $1+3$	10-11. Identity +
12. 4	11-12. Addition

Scoring

Each team earns points for steps that are justified correctly. The team with more points wins.



Cut out the cards and laminate them for repeated use.

ADDITION PROPERTIES		MULTIPLICATION PROPERTIES	
Identity +	$a + 0 = a$	Identity x	$a \times 1 = a$
Inverse +	$a + ^-a = 0$	Inverse x	$a \times \frac{1}{a} = 1$
Commutative +	$a + b = b + a$	Commutative x	$a \times b = b \times a$
Associative +	$(a + b) + c = a + (b + c)$	Associative x	$(a \times b) \times c = a \times (b \times c)$
Definition of subtraction	$a - b = a + ^-b$	Definition of division	$a \div b = a \times \frac{1}{b}$
Distributive property of multiplication over addition		$a(b + c) = ab + ac$	

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Distributive property of multiplication over addition			

$\frac{1}{2} \times 6 \times 2$	Reasons 3	$1 + 2 + 3 + 4 + 5$	Reasons 3
1. $\frac{1}{2} \times 6 \times 2$	1. Given	1. $1 + 2 + 3 + 4 + 5$	1. Given
2. $\frac{1}{2} \times 2 \times 6$	1-2.	2. $(1 + 2) + (3 + 4) + 5$	1-2.
3. $\left(\frac{1}{2} \times 2\right) \times 6$	2-3.	3. $3 + 7 + 5$	1-3.
4. 1×6	3-4.	4. $(3 + 7) + 5$	1-4.
5. 6	4-5.	5. $10 + 5$	1-5.
		6. 15	1-6.

4 - 3(7 - 14)		Reasons (3)	1 × 2 × 3 × 4 × 5		Reasons (4)
1.	$4 - 3(7 - 14)$	1. Given	1.	$1 \times 2 \times 3 \times 4 \times 5$	1. Given
2.	$4 - 21 + 42$	1-2.	2.	$(1 \times 2) \times (3 \times 4) \times 5$	1-2.
3.	$4 + ^{-}21 + 42$	2-3.	3.	$2 \times (3 \times 4) \times 5$	2-3.
4.	$4 + (^{-}21 + 42)$	3-4.	4.	$2 \times 12 \times 5$	3-4.
5.	$4 + 21$	4-5.	5.	$2 \times 5 \times 12$	4-5.
6.	25	5-6.	6.	$(2 \times 5) \times 12$	5-6.
8 - 6 + 2 + 6		Reasons (3)	7.	10×12	6-7.
1.	$8 - 6 + 2 + 6$	1. Given	8.	120	7-8.
2.	$8 + 2 - 6 + 6$	1-2.	$2\left(\frac{1}{6} + 4\right) \div 8$		Reasons (4)
3.	$8 + 2 + ^{-}6 + 6$	2-3.	1.	$2\left(\frac{1}{6} + 4\right) \div 8$	1. Given
4.	$(8 + 2) + (^{-}6 + 6)$	3-4.	2.	$2\left(\frac{1}{6} + 4\right) \times \frac{1}{8}$	1-2.
5.	$10 + (^{-}6 + 6)$	4-5.	3.	$\frac{1}{3} + 8 \times \frac{1}{8}$	2-3.
6.	$10 + 0$	5-6.	4.	$\frac{1}{3} + \left(8 \times \frac{1}{8}\right)$	3-4.
7.	10	6-7.	5.	$\frac{1}{3} + 1$	4-5.
14 + 7(5 - 2)		Reasons (3)	6.	$1\frac{1}{3}$	5-6.
1.	$14 + 7(5 - 2)$	1. Given			
2.	$14 + 35 - 14$	1-2.			
3.	$14 + 35 + ^{-}14$	2-3.			
4.	$14 + ^{-}14 + 35$	3-4.			
5.	$(14 + ^{-}14) + 35$	4-5.			
6.	$0 + 35$	5-6.			
7.	35	6-7.			

$-2+7(3-4)+7$	Reasons 4	$-15-\frac{1}{5}+3\left(5+\frac{2}{5}\right)$	Reasons 5
1. $-2+7(3-4)+7$	1. Given	1. $-15-\frac{1}{5}+3\left(5+\frac{2}{5}\right)$	1. Given
2. $-2+21-28+7$	1-2.	2. $-15-\frac{1}{5}+15+\frac{6}{5}$	1-2.
3. $-2+21+^{-}28+7$	2-3.	3. $-15+15-\frac{1}{5}+\frac{6}{5}$	2-3.
4. $-2+(21+^{-}28)+7$	3-4.	4. $(-15+15)-\frac{1}{5}+\frac{6}{5}$	3-4.
5. $-2+^{-}7+7$	4-5.	5. $0-\frac{1}{5}+\frac{6}{5}$	4-5.
6. $-2+(-7+7)$	5-6.	6. $0+^{-}\frac{1}{5}+\frac{6}{5}$	5-6.
7. $-2+0$	6-7.	7. $0+\left(-\frac{1}{5}+\frac{6}{5}\right)$	6-7.
8. -2	7-8.	8. $0+\frac{5}{5}$	7-8.
8. -2	7-8.	9. 1.	8-9.

$3\times 10\times\frac{1}{3}-8$	Reasons 5
1. $3\times 10\times\frac{1}{3}-8$	1. Given
2. $3\times\frac{1}{3}\times 10-8$	1-2.
3. $\left(3\times\frac{1}{3}\right)\times 10-8$	2-3.
4. $1\times 10-8$	3-4.
5. $(1\times 10)-8$	4-5.
6. $10-8$	5-6.
7. $10+^{-}8$	6-7.
8. 2.	7-8.

$\frac{1}{2} \times 6 \times 2$	Reasons 3
1. $\frac{1}{2} \times 6 \times 2$	1. Given
2. $\frac{1}{2} \times 2 \times 6$	1-2. Commutative x
3. $\left(\frac{1}{2} \times 2\right) \times 6$	2-3. Associative x
4. 1×6	3-4. Inverse x
5. 6	4-5. Identity x

$1+2+3+4+5$	Reasons 3
1. $1+2+3+4+5$	1. Given
2. $(1+2)+(3+4)+5$	1-2. Associative +
3. $3+7+5$	1-3. Addition
4. $(3+7)+5$	1-4. Associative +
5. $10+5$	1-5. Addition
6. 15	1-6. Addition

$4-3(7-14)$	Reasons 3
1. $4-3(7-14)$	1. Given
2. $4-21+42$	1-2. Distributive
3. $4+^{-}21+42$	2-3. Def of subtraction
4. $4+(-21+42)$	3-4. Associative +
5. $4+21$	4-5. Addition
6. 25	5-6. Addition

$8-6+2+6$	Reasons 3
1. $8-6+2+6$	1. Given
2. $8+2-6+6$	1-2. Commutative +
3. $8+2+^{-}6+6$	2-3. Def of subtraction
4. $(8+2)+(-6+6)$	3-4. Associative +
5. $10+(-6+6)$	4-5. Addition
6. $10+0$	5-6. Inverse +
7. 10	6-7. Identity +

$14+7(5-2)$	Reasons 3
1. $14+7(5-2)$	1. Given
2. $14+35-14$	1-2. Distributive
3. $14+35+^{-}14$	2-3. Def of subtraction
4. $14+^{-}14+35$	3-4. Commutative +
5. $(14+^{-}14)+35$	4-5. Associative +
6. $0+35$	5-6. Inverse +
7. 35	6-7. Identity +

$^{-}2+7(3-4)+7$	Reasons 4
1. $^{-}2+7(3-4)+7$	1. Given
2. $^{-}2+21-28+7$	1-2. Distributive
3. $^{-}2+21+^{-}28+7$	2-3. Def of subtraction
4. $^{-}2+(21+^{-}28)+7$	3-4. Associative +
5. $^{-}2+^{-}7+7$	4-5. Addition
6. $^{-}2+(-7+7)$	5-6. Associative +
7. $^{-}2+0$	6-7. Inverse +
8. $^{-}2$	7-8. Identity +

$-15 - \frac{1}{5} + 3\left(5 + \frac{2}{5}\right)$	Reasons 5
1. $-15 - \frac{1}{5} + 3\left(5 + \frac{2}{5}\right)$	1. Given
2. $-15 - \frac{1}{5} + 15 + \frac{6}{5}$	1-2. Distributive
3. $-15 + 15 - \frac{1}{5} + \frac{6}{5}$	2-3. Commutative +
4. $(-15 + 15) - \frac{1}{5} + \frac{6}{5}$	3-4. Associative +
5. $0 - \frac{1}{5} + \frac{6}{5}$	4-5. Inverse +
6. $0 + -\frac{1}{5} + \frac{6}{5}$	5-6. Def of subtraction
7. $0 + \left(-\frac{1}{5} + \frac{6}{5}\right)$	6-7. Associative +
8. $0 + \frac{5}{5}$	7-8. Addition
9. 1	8-9. Identity +

$1 \times 2 \times 3 \times 4 \times 5$	Reasons 4
1. $1 \times 2 \times 3 \times 4 \times 5$	1. Given
2. $(1 \times 2) \times (3 \times 4) \times 5$	1-2. Associative x
3. $2 \times (3 \times 4) \times 5$	2-3. Identity x
4. $2 \times 12 \times 5$	3-4. Multiplication
5. $2 \times 5 \times 12$	4-5. Commutative x
6. $(2 \times 5) \times 12$	5-6. Associative x
7. 10×12	6-7. Multiplication
8. 120	7-8. Multiplication

$2\left(\frac{1}{6} + 4\right) \div 8$	Reasons 4
1. $2\left(\frac{1}{6} + 4\right) \div 8$	1. Given
2. $2\left(\frac{1}{6} + 4\right) \times \frac{1}{8}$	1-2. Def of division
3. $\frac{1}{3} + 8 \times \frac{1}{8}$	2-3. Distributive
4. $\frac{1}{3} + \left(8 \times \frac{1}{8}\right)$	3-4. Associative x
5. $\frac{1}{3} + 1$	4-5. Inverse x
6. $1\frac{1}{3}$	5-6. Addition

$3 \times 10 \times \frac{1}{3} - 8$	Reasons 5
1. $3 \times 10 \times \frac{1}{3} - 8$	1. Given
2. $3 \times \frac{1}{3} \times 10 - 8$	1-2. Commutative x
3. $\left(3 \times \frac{1}{3}\right) \times 10 - 8$	2-3. Associative x
4. $1 \times 10 - 8$	3-4. Inverse x
5. $(1 \times 10) - 8$	4-5. Associative x
6. $10 - 8$	5-6. Identity x
7. $10 + -8$	6-7. Def of subtraction
8. 2	7-8. Addition